INFLUENCE OF THERMAL RESISTANCE OF CONDENSATE DROPS ON HEAT AND MASS TRANSFER IN CONDENSATION OF VAPOR FROM A VAPOR – GAS MIXTURE

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Heat transfer to a solid wall during condensation of a vapor from a vapor–gas mixture in drops is discussed.

In general, the rate of vapor condensation from a vapor–gas mixture is governed by the conditions under which the vapor is transported to the interface and by heat transfer through the liquid phase which forms on the solid surface.

The thermal resistance of the drops during condensation of vapor in drops on a surface is usually neglected [1-3], but, as we show below, under certain conditions this thermal resistance of the condensate drops can significantly affect the rate of heat and mass transfer.

Examining the theoretical papers on the influence of the thermal resistance of condensate drops on heat transfer to a wall, we find an analysis [4] of the simple case corresponding to growth of a hemispherical drop on a wall. Usually, on the other hand, the drops which form on metal surfaces have wetting angles less than 90°, so that the results of [4] are not of general applicability.

Let us examine the process of heat transfer through a single growing liquid drop. If we neglect the thermocapillary and gravitational convection and if we neglect the mixing of the liquid in the drop as a result of its growth, we can describe the temperature in the drop by the Fourier equation, which becomes, in toroidal coordinates,

$$\frac{\partial}{\partial\alpha}\left(\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+\cos\beta}\cdot\frac{\partial T}{\partial\alpha}\right)+\frac{\partial}{\partial\beta}\left(\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+\cos\beta}\cdot\frac{\partial T}{\partial\beta}\right)=\frac{R_0^2}{(\operatorname{ch}\alpha+\cos\beta)^2\,a}\cdot\frac{\partial T}{\partial t}. \tag{1}$$

Toroidal coordinates are convenient in this case of drop growth, in which the interfaces correspond to fixed surfaces $\beta = 0$, $\beta = \beta_0$ (Fig. 1). The time derivatives in the fixed ($\partial/\partial t$) and moving ($\partial'/\partial t$) coordinate systems are related by $\partial/\partial t = \partial'/\partial t + \dot{\alpha}\partial/\partial\alpha + \dot{\beta}\partial/\partial\beta$, where $\dot{\alpha} = \partial\alpha(P)/\partial t$, $\dot{\beta} = \partial\beta(P)/\partial t$, corresponding to a fixed point P, which can be specified by means of the cylindrical coordinates R, z (Fig. 1):

$$R=\frac{R_0\operatorname{sh}\alpha}{\operatorname{ch}\alpha+\cos\beta},\quad z=\frac{R_0\sin\beta}{\operatorname{ch}\alpha+\cos\beta}.$$

Accordingly, $\dot{\alpha}$ and $\dot{\beta}$ are determined from the system of equations dR/dt = 0, dz/dt = 0, and are

$$\dot{\alpha}=f_1(\alpha,\ \beta)\,\dot{R}/R_0,\quad \dot{\beta}=f_2(\alpha,\ \beta)\,\dot{R}_0/R_0.$$

Since $R_0 = R_0(t)$, we have $\partial'/\partial t = \dot{R}_0\partial/\partial R_0$, and Eq. (1) can be written in the following form in the coordinate system tied to the growing drop:

$$\frac{\partial}{\partial\alpha}\left(\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+\cos\beta}\ \frac{\partial T}{\partial\alpha}\right)+\frac{\partial}{\partial\beta}\left(\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+\cos\beta}\ \frac{\partial T}{\partial\beta}\right)=$$

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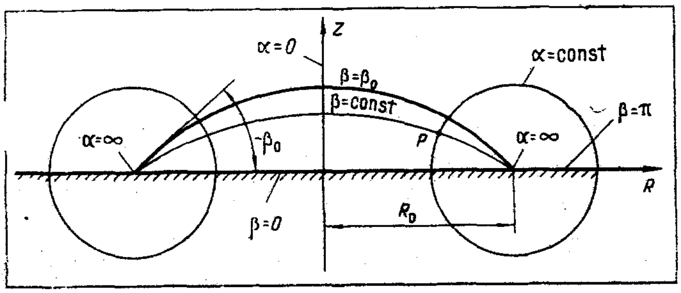

Fig. 1. Calculation model.

$$= \frac{R_0 \dot{R}_0}{a(\operatorname{ch}\alpha - \cos\beta)^2}\left[R_0 \frac{\partial T}{\partial R_0} - f_1(\alpha, \beta)\frac{\partial T}{\partial \alpha} + f_2(\alpha, \beta)\frac{\partial T}{\partial \beta}\right]. \tag{2}$$

As the boundary conditions we assume that the wall temperature is constant,

$$T = T_w = \text{const} \quad \text{for } \beta = 0, \tag{3}$$

and that the heat-transfer coefficient α_1 and the mass-transfer coefficient α_2 for convective transfer at the liquid–gas interface are constant.

$$\frac{\lambda}{h_\beta}\frac{\partial T}{\partial \beta} = \alpha_1(T_\infty - T) + \alpha_2 r[P_\infty - P(T)] \quad \text{for } \beta = \beta_0. \tag{4}$$

At small relative temperature drops at the drop surface $[(T - T_W)/T_W \ll 1]$ the saturation vapor pressure can be approximated by a linear equation:

$$P(T) = P_w + P'_T(T - T_w) \quad [P_w = P(T_w), \quad P'_T = \partial P(T_w)/\partial T].$$

We begin the solution with the quasisteady approximation, for which we set the right side of Eq. (2) equal to zero. This approach is valid for a broad range of condensation conditions. We will later see how the temperature field in the drop is affected by the deviation from steady-state conditions.

The exact solution of the corresponding steady-state problem is

$$T = T_w + \sqrt{\operatorname{ch}\alpha + \cos\beta}\int_0^\infty A(\tau)\operatorname{sh}(\beta\tau)P_{-1/2+i\tau}(\operatorname{ch}\alpha)\,d\tau. \tag{5}$$

The function $A(\tau)$ must be determined from boundary condition (4), but this approach involves the solution of an extremely complicated integral equation. Accordingly, for small wetting angles β_0 we use the asymptotic "narrow-band" method, introducing the constant coordinate $\gamma = \beta/\beta_0$ and rewriting the quasisteady problem as

$$\frac{\partial^2 T}{\partial \gamma^2} + \beta_0^2\left[\frac{1}{2}\frac{1}{\operatorname{ch}\alpha + 1}\frac{\partial}{\partial \gamma}\left(\gamma^2\frac{\partial T}{\partial \gamma}\right) + \frac{\operatorname{ch}\alpha + 1}{\operatorname{sh}\alpha}\frac{\partial}{\partial \alpha}\times\right.$$
$$\left.\times\left(\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha + 1}\frac{\partial T}{\partial \alpha}\right)\right] + \beta_0^4\left\{\frac{1}{4}\left[\frac{1}{(\operatorname{ch}\alpha + 1)^2} - \frac{1}{6}\frac{1}{\operatorname{ch}\alpha + 1}\right]\times\right.$$
$$\left.\times\frac{\partial}{\partial \gamma}\left(\gamma^4\frac{\partial T}{\partial \gamma}\right) + \frac{1}{2}\frac{\operatorname{ch}\alpha + 1}{\operatorname{sh}\alpha}\gamma^2\frac{\partial}{\partial \alpha}\left[\frac{\operatorname{sh}\alpha}{(\operatorname{ch}\alpha + 1)^2}\frac{\partial T}{\partial \alpha}\right]\right\} + \ldots = 0 \tag{6}$$

for $\gamma = 0$, $T = T_W$;

$$\frac{\lambda}{R}\left(\operatorname{ch}\alpha + 1 - \frac{\beta_0^2}{2} + \frac{\beta_0^4}{24} - \ldots\right)\frac{\partial T}{\partial \gamma} = \alpha_1\beta_0(T_\infty - T) + \alpha_2 r\beta_0[P_\infty - P_w - P'_T(T - T_w)] \tag{7}$$

for $\gamma = 1$.

We seek a solution as an expansion in the small parameter β_0:

$$T = T_w + T_0 + \beta_0^2 T_2 + \beta_0^4 T_4 + \ldots \tag{8}$$

For T_0, T_2, and T_4 we find

$$\frac{\partial^2 T_0}{\partial \gamma^2} = 0 \quad (0 < \gamma < 1);$$

$$T_0 = 0\ (\gamma=0),\quad \frac{\partial T_0}{\partial\gamma} + \frac{B}{\operatorname{ch}\alpha+1}T_0 = \frac{c}{\operatorname{ch}\alpha+1}\quad (\gamma=1);$$

$$\frac{\partial^2 T_2}{\partial\gamma^2} = -\frac{\operatorname{ch}\alpha+1}{\operatorname{sh}\alpha}\frac{\partial}{\partial\alpha}\left(\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+1}\frac{\partial T_0}{\partial\alpha}\right) - \frac{1}{2}\frac{1}{\operatorname{ch}\alpha+1}\frac{\partial}{\partial\gamma}\left(\gamma^2\frac{\partial T_0}{\partial\gamma}\right),$$

$$T_2 = 0\ (\gamma=0),\quad \frac{\partial T_2}{\partial\gamma} + \frac{B}{\operatorname{ch}\alpha+1}T_2 = \frac{1}{2}\frac{1}{\operatorname{ch}\alpha+1}\frac{\partial T_0}{\partial\gamma}\quad (\gamma=1);$$

$$\frac{\partial^2 T_4}{\partial\gamma^2} + \frac{1}{2}\frac{1}{\operatorname{ch}\alpha+1}\frac{\partial}{\partial\gamma}\left(\gamma^2\frac{\partial T_2}{\partial\gamma}\right) + \frac{\operatorname{ch}\alpha+1}{\operatorname{sh}\alpha}\frac{\partial}{\partial\alpha}\times$$
$$\times\left(\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+1}\frac{\partial T_2}{\partial\alpha}\right) + \frac{1}{4}\left[\frac{1}{(\operatorname{ch}\alpha+1)^2} - \frac{1}{6}\frac{1}{\operatorname{ch}\alpha+1}\right]\frac{\partial}{\partial\gamma}\times$$
$$\times\left(\gamma^4\frac{\partial T_0}{\partial\gamma}\right) + \frac{1}{2}\frac{\operatorname{ch}\alpha+1}{\operatorname{sh}\alpha}\gamma^2\frac{\partial}{\partial\alpha}\left[\frac{\operatorname{sh}\alpha}{(\operatorname{ch}\alpha+1)^2}\frac{\partial T_0}{\partial\alpha}\right] = 0$$
$$(0<\gamma<1),\quad T_4 = 0\ (\gamma=0);$$
$$\frac{\partial T_4}{\partial\gamma} + \frac{B}{\operatorname{ch}\alpha+1}T_4 = \frac{1}{2}\frac{1}{\operatorname{ch}\alpha+1}\frac{\partial T_2}{\partial\gamma} - \frac{1}{24}\frac{1}{\operatorname{ch}\alpha+1}\frac{\partial T_0}{\partial\gamma}\ (\gamma=1).$$

Here $B = \beta_0(R_0/\lambda)(\alpha_1 + \alpha_2 r P_T')$ and $c = \beta_0(R_0/\lambda)(\alpha_1\Delta T + \alpha_2 r\Delta P)$.

For definiteness we have assumed that the effective external thermal resistance $(\alpha_1 + \alpha_2 r P_T')^{-1}$ is comparable to the thermal resistance of the drop, i.e., that B is on the order of unity.

Accordingly,

$$T_0 = c\Phi_0(\alpha)\gamma,\quad T_2 = c\left[\frac{\Phi_1(\alpha)}{6}\gamma^3 + \Phi_2(\alpha)\gamma\right],$$
$$T_4 = c\left\{\Phi_4(\alpha)\gamma - \frac{1}{2}\frac{1}{\operatorname{ch}\alpha+1}\left[\frac{\Phi_1(\alpha)}{10}\gamma^5 + \frac{\Phi_2(\alpha)}{3}\gamma^3\right] -\right.$$
$$-\frac{\operatorname{ch}\alpha+1}{\operatorname{sh}\alpha}\frac{\partial}{\partial\alpha}\left[\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+1}\left(\frac{\Phi_1'(\alpha)}{120}\gamma^5 + \frac{\Phi_2'(\alpha)}{6}\gamma^3\right)\right] -$$
$$\left.-\frac{1}{20}\left[\frac{1}{(\operatorname{ch}\alpha+1)^2} - \frac{1}{6}\frac{1}{\operatorname{ch}\alpha-1}\right]\Phi_0(\alpha)\gamma^5 - \frac{1}{40}\frac{\operatorname{ch}\alpha+1}{\operatorname{sh}\alpha}\frac{\partial}{\partial\alpha}\left[\frac{\operatorname{sh}\alpha}{(\operatorname{ch}\alpha+1)^2}\Phi_0'(\alpha)\gamma^5\right]\right\}, \tag{9}$$

$$\Phi_0(\alpha) = \frac{1}{\operatorname{ch}\alpha+1+B},\quad \Phi_2(\alpha) = \frac{2(2+B)(\operatorname{ch}\alpha+1)}{(\operatorname{ch}\alpha+1+B)^3} -$$
$$-\frac{1}{(\operatorname{ch}\alpha+1+B)(\operatorname{ch}\alpha+1)} - \frac{\operatorname{ch}\alpha+1}{(\operatorname{ch}\alpha+1+B)^2};\quad \Phi_2(\alpha) =$$
$$= \frac{1}{2}\frac{1}{(\operatorname{ch}\alpha+1+B)^2} - \frac{1}{2}\frac{\operatorname{ch}\alpha+1+B/3}{\operatorname{ch}\alpha+1+B}\Phi_1(\alpha),$$

$$\Phi_4(\alpha) = \frac{1}{\operatorname{ch}\alpha+1+B}\left[\frac{\Phi_1(\alpha)}{2} + \Phi_2(\alpha)\right] + \frac{(\operatorname{ch}\alpha+1)^2}{\operatorname{sh}\alpha(\operatorname{ch}\alpha+1+B)}\frac{\partial}{\partial\alpha}\times$$
$$\times\left[\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+1}\left(\frac{\Phi_1'(\alpha)}{24} + \frac{\Phi_2(\alpha)}{2}\right)\right] + \frac{1}{4}\frac{1}{(\operatorname{ch}\alpha+1+B)^2}\times$$
$$\times\left(\frac{1}{\operatorname{ch}\alpha+1} - \frac{1}{6}\right)\left(1 - \frac{B}{5}\frac{1}{\operatorname{ch}\alpha+1}\right) - \frac{1}{8}\frac{(\operatorname{ch}\alpha+1)^2}{\operatorname{sh}\alpha(\operatorname{ch}\alpha+1+B)}\times$$
$$\times\left(1 + \frac{B}{5}\frac{1}{\operatorname{ch}\alpha-1}\right)\frac{\partial}{\partial\alpha}\left[\frac{\operatorname{ch}\alpha-1}{(\operatorname{ch}\alpha+1)(\operatorname{ch}\alpha+1+B)^2}\right] +$$
$$+\frac{1}{2}\frac{B}{(\operatorname{ch}\alpha+1+B)(\operatorname{ch}\alpha+1)}\left[\frac{\Phi_1(\alpha)}{10} + \frac{\Phi_2(\alpha)}{3}\right] +$$
$$+\frac{B(\operatorname{ch}\alpha+1)}{\operatorname{sh}\alpha(\operatorname{ch}\alpha+1+B)}\frac{\partial}{\partial\alpha}\left[\frac{\operatorname{sh}\alpha}{\operatorname{ch}\alpha+1}\left(\frac{\Phi_1'(\alpha)}{120} + \frac{\Phi_2'(\alpha)}{6}\right)\right] -$$
$$-\frac{1}{24}\frac{1}{(\operatorname{ch}\alpha+1+B^2)};\quad \Phi_j' = \frac{d\Phi_j}{d\alpha}. \tag{10}$$

The heat flux from the drop to the wall can be calculated from

$$Q' = 2\pi\lambda \int_0^\infty \frac{1}{h_\beta} \frac{\partial T}{\partial \beta} R h_\alpha d_\alpha = \frac{2\pi R_0 \lambda}{\beta_0} \int_0^\infty \frac{\partial T}{\partial \gamma} \frac{\operatorname{sh}\alpha d\alpha}{\operatorname{ch}\alpha + 1} \quad (\gamma = 0). \tag{11}$$

Since $h_\alpha = h_\beta$, we have $R = R_0 \operatorname{sh}\alpha/(\operatorname{ch}\alpha + \cos\beta)$ (Fig. 1). Substituting (8) into (11), we find the asymptotic behavior

$$Q = Q_0' + \beta_0^2 Q_2' + \beta_0^4 Q_4' + \ldots,$$

where

$$Q_0' = \frac{2\pi R_0 \lambda c}{\beta_0} \int_0^\infty \frac{\Phi_0(\alpha) \operatorname{sh}\alpha d\alpha}{\operatorname{ch}\alpha + 1}, \quad Q_2' = \frac{2\pi R_0 \lambda c}{\beta_0} \int_0^\infty \frac{\Phi_2(\alpha) \operatorname{sh}\alpha d\alpha}{\operatorname{ch}\alpha + 1},$$

$$Q_4' = \frac{2\pi R_0 \lambda c}{\beta_0} \int_0^\infty \frac{\Phi_4(\alpha) \operatorname{sh}\alpha d\alpha}{\operatorname{ch}\alpha + 1}, \ldots$$

Using (10), we find

$$Q' = \frac{2\pi R_0 \lambda c}{\beta_0 B} [q_0(B) + \beta_0^2 q_2(B) + \ldots],$$

where

$$q_0(B) = \ln \frac{2+B}{2}; \; q_2(B) = \frac{2}{3B} \ln \frac{2+B}{2} - \frac{16+6B+B^2}{12(2+B)^2}, \ldots \tag{12}$$

The heat flux to the planar surface covered by a drop of radius R_0 is

$$Q_p = \pi R_0^2 (\alpha_1 \Delta T + \alpha_2 r P_T' \Delta P) = \frac{\pi c \lambda R_0}{\beta_0},$$

so that we have the ratio

$$\frac{Q'}{Q_p} = 2 \frac{q_0(B) + \beta_0^2 q_2(B)}{B}. \tag{13}$$

Calculations show that as B increases the ratio Q'/Q_p decreases, falling below one at $B > 0.4$. Accordingly, the thermal resistance of the condensate drop can affect the heat transfer to the wall only if $B > 0.4$. If $B < 0.4$, the ratio Q'/Q_p becomes larger than one, so that heat and mass transfer can be intensified.

Results calculated from Eq. (13) are shown in Fig. 2. Curve 1, plotted on the basis of two terms for $\beta_0 = \pi/4$, differs only slightly from curve 2, corresponding to the ideal case $\beta_0 \to 0$, i.e., from the result calculated on the basis of the first term in the asymptotic expansion. Obviously, then, for wetting angles $\beta_0 \ll \pi/4$ it is sufficiently accurate for practical purposes to calculate the heat fluxes on the basis of the first iteration alone, in the sense of the "narrow-band" asymptotic behavior.

The part of the heat flux due to vapor condensation is

$$Q'' = Q^{V} - Q^{VI},$$

where

$$Q^{V} = 2\pi\alpha_2 r \Delta P \int_0^\infty R h_\alpha d\alpha; \; Q^{VI} = 2\pi\alpha_2 r P_T' \int_0^\infty (T - T_w) R h_\alpha d\alpha.$$

However, the heat flux Q' found above can be written

$$Q' = Q^{III} - Q^{IV}, \; Q^{III} = 2\pi(\alpha_1 \Delta T + \alpha_2 r \Delta P) \int_0^\infty R h_\alpha d\alpha,$$

$$Q^{IV} = 2\pi(\alpha_1 + \alpha_2 r P_T') \int_0^\infty (T - T_w) R h_\alpha d\alpha.$$

We thus have

$$Q^{VI} = \frac{\alpha_2 r P_T'}{\alpha_1 + \alpha_2 r P_T'} Q^{IV} = \frac{\alpha_2 r P_T'}{\alpha_1 + \alpha_2 r P_T'} (Q^{III} - Q'),$$

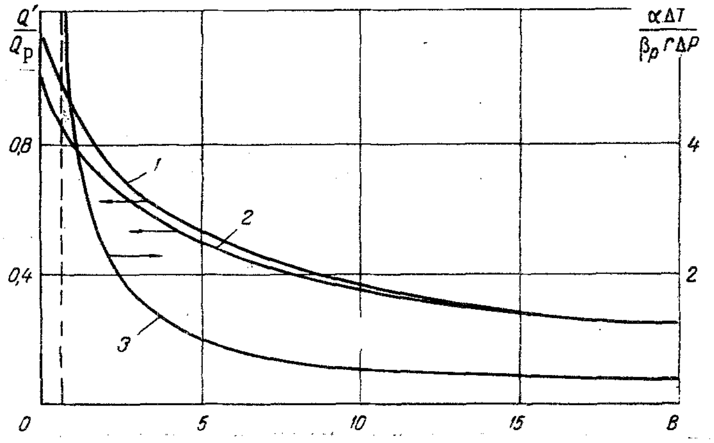

Fig. 2. The ratio Q'/Q_p and the equilibrium ratio of the temperature and pressure drops ΔT and ΔP as functions of the dimensionless parameter B. 1) $\beta_0 = \pi/4$; 2) $\beta \to 0$; 3) $(\alpha_1\Delta T)/(\alpha_2 r\Delta P)$.

since

$$h_\alpha = R_0/(\operatorname{ch}\alpha + \cos\beta), \quad \int_0^\infty (Rh_\alpha)_{\gamma=1}\,d\alpha = \frac{R_0^2}{1+\cos\beta_0}.$$

Finally, we find

$$Q'' = Q_m - \frac{\alpha_2 r P'_T}{\alpha_1 + \alpha_2 r P'_T}\left(\frac{\alpha_1\Delta T + \alpha_2 r\Delta P}{\alpha_2 r\Delta P}\, Q_m - Q'\right), \tag{14}$$

$$Q_m = \frac{2\pi R_0^2\alpha_2 r}{1+\cos\beta_0}\,\Delta P.$$

If the drop grows as a result of condensation alone (i.e., if the drop does not merge with other drops in the time interval under consideration, we can use the expression for the drop volume,

$$\Omega_K = (\pi R_0^3/3\sin^3\beta_0)(1-\cos\beta_0)^2(2+\cos\beta_0),$$

to find

$$\frac{dR_0}{dt} = \frac{Q''\sin^3\beta_0}{\pi R_0^2(1-\cos\beta_0)^2(2+\cos\beta_0)\,r\rho}. \tag{15}$$

Accordingly, the drop stops growing when the parameter B corresponding to its base radius R_0 becomes equal to the root B_* of the function Q''/R_0^2. This value of the radius and that of the corresponding value of the parameter B can be called the equilibrium values, since a drop having reached this size is in dynamic equilibrium with the moist gas moving around it ($dR_0/dt = 0$) (we are not considering the mechanical equilibrium of the drop on a wall in a gas flow). In other words, the rate of evaporation from the central part of the drop surface becomes equal to the rate of condensation at the peripheral part.

Instead of determining B_* through a solution of the complicated transcendental equation we can find the ratio $P'_T\Delta T/\Delta P$ at which the given value of B becomes the equilibrium value:

$$\frac{P'_T\Delta T}{\Delta P} = \frac{A_2(\beta_0 B)}{A_1(\beta_0 B)}; \quad A_1 = \frac{1}{1+\cos\beta_0} - \frac{q_0(B) + \beta_0^2 q_2(B)}{B} + \ldots,$$

$$A_2 = \frac{1}{1+\cos\beta_0} + \frac{\alpha_2 r P'_T}{\alpha_1}\,\frac{q_0(B) + \beta_0^2 q_2(B) + \ldots}{B}.$$

Since we have $\alpha_2 r P'_T \gg 1$ in cases of practical interest, we find a ratio $P'_T\Delta T/\Delta P \gg 1$. In a first approximation we can write

$$\frac{P'_T\Delta T}{\Delta P} = \frac{\alpha_2 r P'_T}{\alpha_1}\,\frac{\ln(1+B/2)}{B/2 - \ln(1+B/2)}. \tag{16}$$

Curve 3 in Fig. 2 is plotted from this equation.

It was shown above that in the range of wetting angles β_0 of interest here it is sufficient to use the first iteration of the "narrow-band" asymptotic approach to solve the quasisteady equation. Accordingly, we also use this iteration to examine the relaxation of the temperature field in the drop due to the finite specific heat of the liquid. In the limit $\beta_0 \to 0$ we have $f_1 \approx -\mathrm{sh}\,\alpha$ and $f_2 \approx -\beta_0\gamma\,\mathrm{ch}\,\alpha$, and Eq. (2) can be approximated by

$$\frac{\partial^2 T}{\partial \gamma^2} = \frac{\beta_0^2 R_0 \dot{R}_0}{a\,\mathrm{sh}\,\alpha\,(\mathrm{ch}\,\alpha + 1)}\left(R_0\frac{\partial T}{\partial R_0} - \mathrm{sh}\,\alpha\frac{\partial T}{\partial \alpha} - \gamma\,\mathrm{ch}\,\alpha\frac{\partial T}{\partial \gamma}\right). \tag{17}$$

Under the assumption that the change in the enthalpy in the drop is due primarily to the change in the enthalpy in the liquid in the course of the phase transition, we seek the temperature field in a drop of radius R_0, and we seek the heat flux Q', as the expansions

$$T = T_0 + \varepsilon T_1 + \ldots,\quad \dot{R}_0 = \dot{R}_{00} + \varepsilon \dot{R}_{01} + \ldots,\quad Q' = Q_0' + \varepsilon Q_1' + \ldots,$$

where T_0, R_{00}, and Q_0' correspond to the quasisteady approximation. In the limit $\beta_0 \to 0$ we find from (12)-(15)

$$\dot{R}_{00} = \frac{4}{3}\frac{\alpha_2 \Delta P}{\beta_0 \rho}\left[\Lambda(1-\theta) + 2(1-\Lambda\theta)\frac{1}{B}\ln\left(1+\frac{B}{2}\right)\right], \tag{18}$$

where $\Lambda = \alpha_1/(\alpha_2 r P_T')$ and $\theta = P_T'\Delta T/\Delta P$.

Choosing the quantity $\varepsilon = (4\alpha_2\beta\Delta P R_0)/(3\rho a)$ as a small parameter and using Eq. (9) for T_0 we find

$$\frac{\partial^2 T_1}{\partial \gamma^2} = -\frac{BC\gamma\,\mathrm{ch}\,\alpha}{\mathrm{sh}\,\alpha\,(\mathrm{ch}\,\alpha + 1)(\mathrm{ch}\,\alpha + 1 - B)^2}. \tag{19}$$

The component T_1 must satisfy the boundary conditions

$$T_1 = 0 \text{ for } \gamma = 0,\quad \frac{\partial T_1}{\partial \gamma} + \frac{B}{\mathrm{ch}\,\alpha + 1}T_1 = 0 \text{ for } \gamma = 1.$$

We thus find

$$T_1 = c\psi_1(\alpha)\gamma - \frac{BC\,\mathrm{ch}\,\alpha}{\mathrm{sh}\,\alpha\,(\mathrm{ch}\,\alpha + 1)(\mathrm{ch}\,\alpha + 1 + B)^2}\cdot\frac{\gamma^3}{6};$$

$$\psi_1(\alpha) = \frac{B\,\mathrm{ch}\,\alpha\,[3(\mathrm{ch}\,\alpha + 1) + B]}{6\,\mathrm{sh}\,\alpha\,(\mathrm{ch}\,\alpha + 1)(\mathrm{ch}\,\alpha + 1 + B)^3};$$

$$Q_1' = \frac{2\pi R_0 \lambda c}{\beta_0}J;\quad J = \int_0^\infty \frac{\psi_1(\alpha)\,\mathrm{sh}\,\alpha\,d\alpha}{\mathrm{ch}\,\alpha + 1};$$

$$J = \frac{B-2}{6B^2}J_1 + \frac{B-1}{6B}(J_2 + 2BJ_3) - \frac{2B-3}{9B^2};$$

$$J_j = \int_0^\infty \frac{d\alpha}{(\mathrm{ch}\,\alpha + 1 + B)^j};$$

$$J_1 = \frac{1}{\sqrt{B(2+B)}}\ln\frac{\sqrt{2+B}+\sqrt{B}}{\sqrt{2+B}-\sqrt{B}},\quad J_2 = \frac{(1+B)J_1 - 1}{B(2+B)},$$

$$J_3 = \frac{[2(1+B)^2 + 1]J_1 - 3(1+B)}{2B^2(2+B)^2}.$$

In a determination of the steady-state temperature field, Ivanov [5] assumed that the isothermal surfaces are a family of spheres passing through the drop contour. That this assumption is incorrect can be seen by writing the problem in a toroidal coordinate system, i.e., by using Eq. (4) without its right side. This assumption is equivalent to the assumption $T = \Phi(B)$. Substituting the function $T = \Phi B$ into the steady-state equation corresponding to Eq. (1) we find that it can be solved only in the trivial case $\Phi(B) = \text{const}$. The function $T = \Phi(B)$ can be thought of as the first iteration of the "narrow-band" asymptotic approach for small wetting angles β_0 and for boundary conditions of the first kind. However, the solution of the problem formulated in this manner, which corresponds to the limit $B \to \infty$, $c/B = \Delta T < \infty$, examined above, cannot be used to determine the heat fluxes or the drop-growth rate, since in this formulation we would find $Q' = \infty$ [the function $Q'(B)$ has a logarithmic singularity at the point $B = \infty$]. The solution given in [5] does not satisfy the boundary conditions formulated there.

NOTATION

λ, a, ρ	are the thermal conductivity, thermal diffusivity, and density, respectively, of the liquid phase;
α_1, α_2	are the convective heat- and mass-transfer coefficients;
r	is the latent heat of vaporization;
R_0	is the radius of the drop base;
α, β	are the toroidal coordinates tied to the growing drop;
h_α, h_β	are the Lamé coefficients;
t	is the time;
β_0	is the wetting angle;
T_W	is the wall temperature;
T_∞	is the temperature of the vapor – gas mixture far from the wall;
P_∞	is the vapor pressure far from the wall;
$P(T)$	is the saturation vapor pressure at temperature T;
Q'	is the heat flux from the drop to the wall;
Q_p	is the heat flux to the plane surface covered by a drop of radius R_0;

$\Delta T = T_\infty - T_W$; $\Delta P = P_\infty - P_W$; $P_W = P(T_W)$.

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